The Sun and the Moon a Riddle in the Sky

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Abstract: Both the sun and the full moon images are uniform, though the light density at the sun surface is uniform and not uniform at the moon surface. Why? The paradox is discussed in terms of Lambertian Scattering.

The following two images in fig.-1 show the sun and the full-moon:

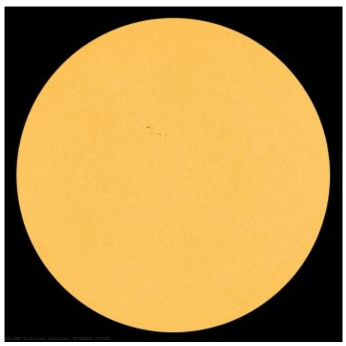

Fig.-1: The sun,[1] and the full moon,[2] images. Both have a uniform light distribution.

The sun photo shows uniform distribution of light intensity within the image from end to end. There are some details of mountains and dry lakes in the moon photo, but apart from that, the light distribution is also uniform within the moon image.

The radiance, or brightness of a point on a Lambertian surface is constant with respect to the angle between the light direction and the normal to the surface,[3-4] . The sun is a Lambertian light source and the light density at the sun surface, Is, is uniform. Therefore, according to Lambert's cosine law the sun image will be uniform, as seen in fig.-1 (left).

This is not the case with the full-moon, fig.-1 (right). The moon light is scattered sunlight. At its center the moon surface is perpendicular to the illuminating sunlight and it becomes more and more inclined to it toward the periphery. Therefore, the sunlight density will be maximal at the moon center, and will fade away to zero at its periphery.

Considering an angle θ between the sun's line of sight to the moon, and a line from the moon center to a point on its surface, the light density at that point will be proportional to $Im * \cos(\theta)$, where *Im* is the sunlight density at the moon center. Thus it will vary from *Im* at the center toward zero at the periphery.

For this reason Lambertian scattering of sunlight by the moon will yield a non-uniform moon image, most bright at the moon center, and fading to zero toward the periphery. This is not the case as seen in fig.-1 (right), the moon image is uniform as is the sun image.

Therefore, it is taken for granted by everybody that the moon surface is not Lambertian, and models of non-lambertian scattering are looked to explain the uniform appearance of the moon image. For example, a model of surface roughness,[5] and a model of retroreflective dust balls,[6] are suggested.

It may be speculated that the moon surface has some special property that makes it non-Lambertian. However, photos of all the planets, taken from space with the sun at the back of the observer, show similar uniformity, [7]. Mars is similar to the moon, but the Earth is different and Venus is covered with heavy clouds. So it is unlikely that the effect comes from surface material properties.

The purpose of this discussion is to indicate that the uniformity of the moon image is a direct consequence of Lambertian scattering. Further mechanisms or models are not necessary. The moon surface is Lambertian after all.

Consider an imaginary line between the sun and the earth, or between the sun and the moon, and a unit cross section area *a* on a plane perpendicular to that line.

Fig.-2 shows an observer on the earth looking directly at the sun (left), and looking at night at the full moon, while the sun is behind his back (right).

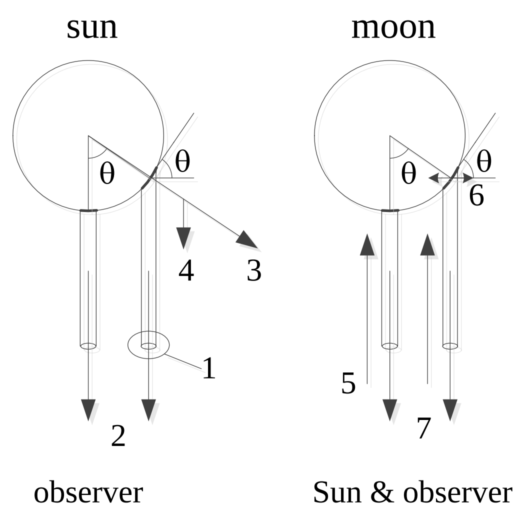

Fig.-2: Relative configurations of the sun, the moon and an observer on the earth.

(1) Cross section area *a*.
(2) The sunlight direction to an observer on the earth.
(3) Equivalent mean direction of sun light from the sun surface.
(4) The component of sunlight observed on earth.
(5) Sunlight toward the moon.
(6) Polarization of the of surface scattering dipoles
(7) Moonlight scattered back to the earth.

Sunlight is a Lambertian source of light emitted uniformly from its surface, fig.-2 (left). That is, each point on the surface emits equal light in all the directions of the solid angle 2π, and the mean equivalent light, fig.-2 (3), is perpendicular to the sun surface.

The angle θ in fig.-2 defines the position of any point on the sun surface, or on the moon surface. The area on the sun surface observed through the cross section area *a* will be:

$$a \,/\, \mathrm{Cos}(\theta), \qquad (1)$$

thus, it is equal to *a* at the sun center, and it increases toward the periphery. The mean light emitted from that area is perpendicular to the sun surface, fig.-2 (3), and its component seen by an observer on earth, fig.-2 (4), will be proportional to Cos(θ).

Therefore the light observed through the cross section *a* will be proportional to:

$$Is * (a \,/\, \mathrm{Cos}(\theta)) * \mathrm{Cos}(\theta). \qquad (2)$$

Hence, the light toward the observer does not depend on Cos(θ) and the sun brightness observed from earth will be uniform,[3-4] .

The case of the moon, fig.-2 (right), is discussed in a similar manner. However, moonlight is a scattered sunlight, and the sunlight on the moon surface, fig.-2 (5), is Im * Cos(θ).

Repeating the same argument, the full moonlight would be proportional to:

$$Im * (a / \mathrm{Cos}(\theta) * \mathrm{Cos}(\theta) * \mathrm{Cos}(\theta)). \qquad (3)$$

The middle Cos(θ) in the Eq. (3) comes from the fading light intensity on the moon. So that moon image should also fade to zero from its center toward its periphery. This is not the case as may be observed by one look at the full moon.

It is beneficial to go a little deeper into the scattering mechanism. Sunlight that falls on the moon, fig.-2 (5), induces dipole charge oscillations on its face, fig.-2 (6). Each such a dipole is a source of light, and the overall scattered light is the combined effect of all the dipoles.

Each dipole oscillates in the electric field direction of the coming sunlight, that is, in a plane perpendicular to it, and the maximal emission of a dipole is perpendicular to its oscillation, that is, back to the sun, fig.-2 (7).

Therefore, the mean equivalent moonlight will also be directed back to the sun, and in full moon also to an observer on the earth. It will not be perpendicular to the moon surface, as was the case with the sun. There is no need for the right Cos(θ) of Eq. (2) in Eq. (3).

Repeating the argument with fading sunlight on the moon surface, the back scattered moonlight will be proportional then to:

$$(a / \mathrm{Cos}(\theta)) * \mathrm{Cos}(\theta), \qquad (4)$$

and an observer will see a uniform full moon.

Similar effect may be observed in photos of a tennis ball. The photos of fig.-3 are taken with a phone camera so that they may be repeated by anyone.

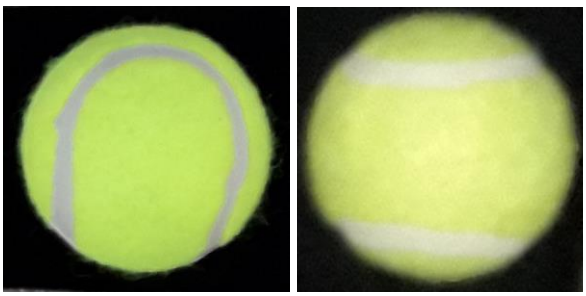

Fig.3: A tennis ball image taken at dawn (left), and with a camera flash in a dark room (right).

Fig.3 (left), was taken from above at dawn, when there is no direct sunlight and indirect light illuminates the ball from all the directions. Therefore the mean scattered light from the ball, at any point on it, will be perpendicular to its surface. This configuration resembles the sun and the observed uniform ball image is similar to the sun image of fig.1 (left).

Fig.-3 (right), was taken with the camera flash in a dark room, with the camera located about 1.5 meters above the ball. This configuration resembles full moon back scattering. The image is somewhat stronger at the center, but the light density is not far away from uniformity, it does not obey the cosine law of eq. (3). In this sense the image is similar to that of the full moon of fig-1 (right). Thus, images on earth of a tennis ball are similar to the real sun and moon images. They indicate that the Lambert scattering mechanism is responsible to the image uniformity. Possible deviation from non-uniformity may be due to fluorescence of the ball paint.

In summary, the mean Lambertian back scattering from the full moon is directed back to the sun because the scattering dipoles on the moon oscillate in a plane perpendicular to the coming sunlight. This polarization stems from the electro-magnetic nature of light and Maxwell's equations. Any calculation that does not take it into account will not be correct.

Lambertian back scattering yields a uniform moon image.

Lambertian back-scattering from any surface will not depend on the surface inclination angle, and the surface will look uniform. Deviation from uniformity may be due to non-Lambertian scattering, partial specular reflection, shadowing, multiple scattering where the polarization direction of the scattering dipoles is lost, etc.

[1] Sun image https://sdo.gsfc.nasa.gov/assets/img/latest/latest_1024_HMIIF.jpg

[2] Moon image http://www.newsfour.ie/wp-content/uploads/2018/02/moon-1.jpg

[3] W. J. Smith, *Modern Optical engineering*, 3rd Ed., (McGrow-Hill, NY, 2000), pp. 221-222.

[4] F. L. Pedrotti, L. S. Pedrotti, *Introduction to Optics*, 2nd Ed. (Prentic-Hall, NJ, 1993), pp. 11-12.

[5] M. Oren, S. K. Nayar, *Generalization of Lambert's reflectance model*, in (Proc. of ACM SIGGRAPH 94, 1994), pp. 239–246, and references therein.

[6] Basic Principles of Surface Reflectance, https://www.cs.cmu.edu/afs/cs/academic/class/15462-f09/www/lec/lec8.pdf

[7] True-Color Photos of All the Planets https://owlcation.com/stem/True-Color-Photos-of-All-the-Planets

The riddle part on the net: July, 2011. The solution on the net: July 2018.

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