

The Sun and the Moon a Riddle in the Sky

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Abstract: Both the sun and the full moon images are uniform, though the light density at the sun surface is uniform and not uniform at the moon surface. Nevertheless, the uniform appearance of the full moon is a direct consequence of Lambertian Scattering.

The sun image is uniform except some marginal blur at its periphery. The full-moon image shows some details of mountains and dry lakes, but apart from that the moon image is uniform as that of the sun.

The following two images in fig.-1 show the sun and the full-moon:

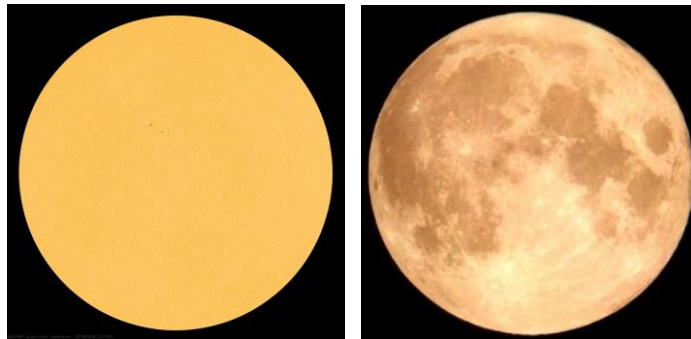


Fig.-1: The sun¹ and the full moon² images. Both have a uniform brightness distribution.

The sun is a Lambertian light source, that is, each point on its surface scatters light equally in all directions. The moon scatters the sun light that falls on its surface, and according to Lambert's cosine law it will be most bright at its face center, and the light intensity will fade to zero when moving toward its periphery.

However, as fig.-1 shows, and as is seen by the eye each month, the full-moon image is uniform, and theories of non-Lambertian light scattering of the moon surface have been suggested in order to explain the observation^{3,4}.

Photos of all the planets and the planet moons, taken from space with the sun at the back of the observer, show similar uniformity⁵. Mars is similar to the moon, but Venus is covered with heavy

clouds and the sunlight is scattered from a gas phase.

Fig.-2 shows an image of the earth taken from the moon ⁶:



Fig.-2: The earth image taken from the moon

The image includes large areas of solid land, liquid water and gas clouds, and the uniformity is observed with each of them. It is, therefore, unlikely that the observed uniformity is the outcome of some property of the scattering medium. Models of light scattering that correspond to this case, deal only with solid surfaces. Explanation or models that will work for scattering from the three phases are difficult to find.

In addition to the full-moon, there are many images of spheres and other curved surfaces with similar directional illumination, but true photos of any of them, that obeys Lambert's cosine law, are again difficult to find. It seems that all the photos that obey the law are at least partly simulated.

When an electromagnetic wave advances in a medium it polarizes its matter, and each polarized dipole is a source of electromagnetic wave. The effect of all the dipoles is calculated, for example, by vector addition of their electric fields. Each dipole has a different phase that depends on its position, and if the material is uniform these fields cancel each other in all the directions except the forward direction, where the effect is refraction.

Scattering comes from non-uniform fluctuations of material density, and then scattered waves may advance in any direction. The intensity of scattered light is the sum of the intensities of the scattering centers and it does not depend on the phases of the scattering dipoles.

The purpose of this discussion is to indicate that the uniformity of the moon image is a direct consequence of Lambertian scattering. Further mechanisms or models are not necessary. The moon surface is Lambertian after all.

Consider an imaginary line between the sun and the earth, or between the sun and the moon, and a unit cross section area a on a plane perpendicular to that line.

Fig.-3 shows an observer on the earth looking directly at the sun (left), and looking at night at the full moon, while the sun is behind his back (right).

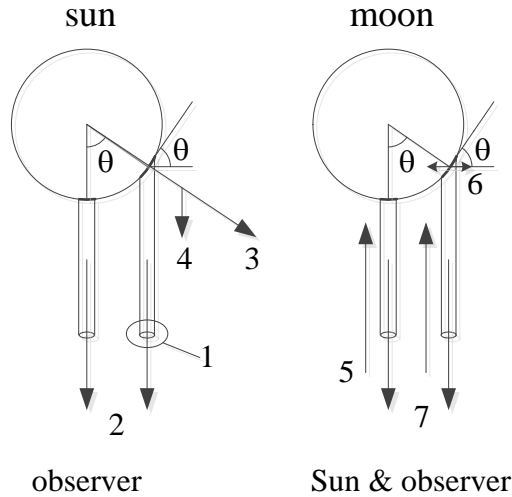


Fig.-3: Relative configurations of the sun, the moon and an observer on the earth.

- (1) Cross section area a .
- (2) The sunlight direction to an observer on the earth.
- (3) Equivalent mean direction of sun light from the sun surface.
- (4) The component of sunlight observed on earth.
- (5) Sunlight toward the moon.
- (6) Polarization of the of surface scattering dipoles
- (7) Moonlight scattered back to the earth.

θ is the angle between a line from the sun center to an observer on the earth, and a line between the sun center and any point on its surface.

The sun is a Lambertian source of light emitted uniformly from its surface, fig.-3 (left). That is, each point on the surface emits equal light in all the directions of the solid angle 2π , and the mean equivalent light, fig.-3 (3), is perpendicular to the sun surface.

θ in fig.-3, is the angle between a line from the sun center to an observer on the earth, and a line between the sun center and any point on its surface. θ defines the position of any point on the sun surface. θ similarly defines any position on the moon surface.

Consider a line between the sun and the earth, or between the sun and the moon, and a unit cross section area a on a plane perpendicular to that line.

The area on the sun surface observed through the cross section area a will be:

$$a / \text{Cos}(\theta), \quad (1)$$

thus, it is equal to a at the sun center, and it increases toward the periphery. The mean light emitted from that area is perpendicular to the sun surface, fig.-3 (3), and its component seen by an observer on earth, fig.-3 (4), will be proportional to $\text{Cos}(\theta)$.

Therefore the sun-light observed through the cross section a will be proportional to:

$$I_s * (a / \text{Cos}(\theta)) * \text{Cos}(\theta). \quad (2)$$

Where I_s is the sunlight density at its surface. Hence, the light toward the observer does not depend on $\text{Cos}(\theta)$ and the sun brightness observed from earth will be uniform⁷⁻⁸.

The case of the moon, fig.-3 (right), is discussed in a similar manner. However, moonlight is a scattered sunlight, and the sunlight on the moon surface, fig.-3 (5), is $I_m * \text{Cos}(\theta)$, where I_m is the sunlight density at the moon face center.

Repeating the same argument, the full moonlight would be proportional to:

$$I_m * \text{Cos}(\theta) * (a / \text{Cos}(\theta)) * \text{Cos}(\theta). \quad (3)$$

The left $\text{Cos}(\theta)$ in the Eq. (3) comes from the fading light intensity on the moon. So, by Eq. (3), the moon image should also fade to zero from its face center toward its periphery. This is not the case as observed in fig.-1 (right), or by looking at the full moon.

It is beneficial to go a little deeper into the scattering mechanism. Sunlight that falls on the moon, fig.-3 (5), induces dipole charge oscillations on its face, fig.-3 (6). Each such a dipole is a source of light, and the overall scattered light is the combined effect of all the dipoles.

Each dipole oscillates in the electric field direction of the coming sunlight, that is, in a plane perpendicular to it, and the maximal emission of a dipole is perpendicular to its oscillation, that is, back to the sun, fig.-3 (7).

Therefore, the mean equivalent moonlight will also be directed back to the sun, and in full moon also to an observer on the earth. It will not be perpendicular to the moon surface, as was the case with the sun surface. There is no need for the right $\text{Cos}(\theta)$ of Eq. (2) in Eq. (3).

Repeating the argument with fading sunlight on the moon surface, the back scattered moonlight will be proportional then to:

$$(a / \cos(\theta)) * \cos(\theta), \quad (4)$$

and an observer on earth will see a uniform full moon.

In summary, the mean Lambertian back scattering from the full moon is directed back to the sun because the scattering dipoles on the moon oscillate in a plane perpendicular to the coming sunlight. Any calculation that does not take the direction of the polarizing dipoles into account will not be correct.

Lambertian back scattering yields a uniform moon image.

Lambertian back-scattering from any surface will not depend on the surface inclination angle to the coming light, and a curved surface will look uniform. It is, therefore, not surprising that true images, that obey Lambert's cosine law, are difficult to find.

In the calculation of Lambert's cosine law there is a hidden assumption that the scattering dipoles oscillate in all random directions in space. This is not the case with the moon, and probably with other examples.

Deviation from uniformity may be due to non-Lambertian scattering, partial specular reflection, shadowing, multiple-scattering where the polarization direction of the scattering dipoles is lost, etc.

References

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- ⁵ True-Color Photos of All the Planets <https://owlcation.com/stem/True-Color-Photos-of-All-the-Planets>
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